University of California, Berkeley
Physics H7A Fall 1998 (*Strovink*)

FINAL EXAMINATION

Directions. Do all six problems (weights are indicated). This is a closed-book closed-note exam except for three $8\frac{1}{2} \times 11$ inch sheets containing any information you wish on both sides. You are free to approach the proctor to ask questions – but he or she will not give hints and will be obliged to write your question and its answer on the board. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Box or circle your answer.

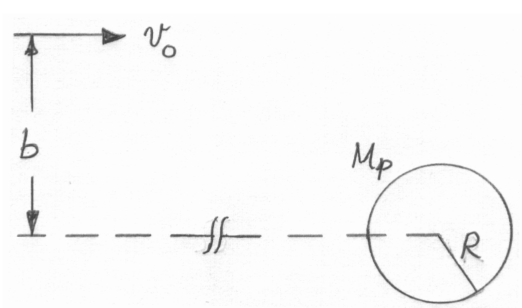
Problem 1. (30 points)
Northern Canada has two peculiar features: owing to lack of roads, most surface transportation occurs by train; and the principal fauna are tiny black flies.

Consider the *elastic* (kinetic energy conserving) head-on collision of a locomotive of mass M and velocity V with a stationary black fly of mass m. You may make any reasonable approximation concerning the relative magnitude of M and m.

a. (15 points)
With what velocity v does the fly recoil from the locomotive?

b. (15 points)
Assuming that the (coasting) locomotive has flat frontal area A, and there are N black flies per cubic meter hovering over the track, apply the results of part (**a.**) to obtain a differential equation for V (neglect air resistance). Solve it to obtain $V(t)$.

Problem 2. (20 points)



A space probe is launched with initial velocity v_0 and impact parameter b toward a very distant planet of radius R and very large mass M_p (see figure). Find the maximum value of b for which the rocket will hit the planet.

Problem 3. (30 points)
For decades, scientists have been designing a "space colony" in which thousands of people could exist while orbiting the sun. People would live on the inside curved surface of a large air-filled cylinder (length of order 10 km, radius R of order 1 km). The cylinder would rotate about its axis with an angular velocity ω such that earth's gravitational acceleration g would be simulated by the centrifugal force acting near that surface. The curved surface would have dirt for farming, and also housing, factories, parks, hills, streams, a lake, etc. Sunlight would enter through one end; it would be controlled by mirrors and shutters to simulate day and night. There would be clouds and weather, etc.

a. (5 points)
Find the angular frequency ω of rotation.

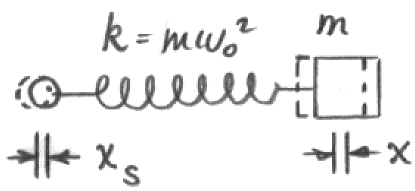
b. (10 points)
Although many aspects of life in this colony would resemble life on earth, one peculiar feature would be the large Coriolis acceleration. When $\mathbf{v}$ (as seen by a colony inhabitant) is perpendicular to $\boldsymbol{\omega}$, the magnitude of the Coriolis acceleration a_C can be expressed as

$$a_C = g\frac{v}{v_C}$$

where v_C is a characteristic velocity. Find v_C appropriate to the surface inhabited by the colonists.

c. (15 points)
For residents in the colony, north is defined to be in the direction of $\boldsymbol{\omega}$; if a resident faces north her right hand points east. A baseball pitcher, new to the colony, fires a ball toward the west with velocity v at his target a distance D away. If he were on the surface of the earth (where the Coriolis force is negligible), the ball would hit its target. In what direction (high, low, north, or south) does the ball miss its target? By what distance d does it miss (you may assume $d \ll D$)?

Problem 4. (30 points)
A mass m is connected by a massless spring of stiffness $k = m\omega_0^2$ to a point of support x_s. When the spring is relaxed, and $x_s = 0$, the mass is at its equilibrium position $x = 0$. The mass moves only in the x direction, without friction.



Suppose that the point of support is constrained by an external force to obey the following motion: $x_s = mA\sin\omega t$, where A and ω are constants, and ω is not necessarily equal to ω_0. The external force does not act directly on the mass, but it nevertheless influences the mass because of the spring.

a. (15 points)
Find the particular solution $x_p(t)$ which would vanish if A were zero.

b. (15 points)
Find the solution $x_0(t)$ which would be correct if the mass were fixed at its equilibrium position and released at $t = 0$.

Problem 5. (30 points)
Consider a thin cylindrical pipe of length L, closed at both ends. The air inside the pipe can support longitudinal (sound) waves that propagate along the axis of the pipe. Let $\xi(x,t)$ be the displacement (along the axis of the pipe) of an air molecule whose equilibrium coordinate (along the same axis) is x. As usual, ξ satisfies the wave equation

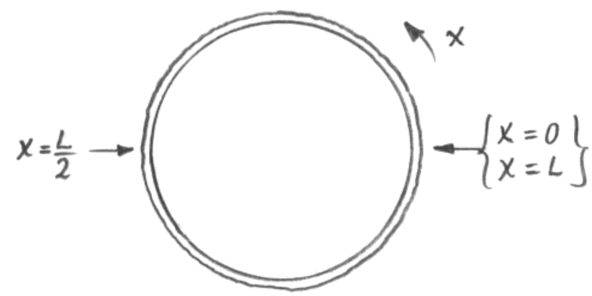
$$\frac{\partial^2 \xi}{\partial x^2} + \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = 0$$

where c is the (phase and group) velocity of sound waves in air.

a. (3 points)
Keeping in mind that the pipe is closed at both ends, write down the boundary conditions on $\xi(0,t)$ and $\xi(L,t)$.

b. (12 points)
The air inside the straight closed pipe is observed to carry a *standing* sinusoidal sound wave. What is the lowest angular frequency ω_s with which this wave can vibrate?

c. (3 points)
The ends of the pipe are now opened and the pipe is bent into a hoop. The end at $x = 0$ is welded to the end at $x = L$, so that the pipe forms a continuously hollow circular torus (like a hula hoop) with a circumference equal to L.



Continue to consider sound waves that propagate along the (bent) axis of the pipe. As long as the circumference of the hoop is much larger than the pipe thickness, which is the case here, $\xi(x,t)$ satisfies the same wave equation as before. However, since the pipe is now bent into a continuously hollow torus, $x = 0$ and $x = L$ now describe the *same* coordinate along the pipe's axis. More generally, $\xi(x,t)$ and $\xi(x+L,t)$ describe the displacement from equilibrium of the *same* molecule.

In light of the above, write down the relationship between $\xi(x,t)$ and $\xi(x+L,t)$.

d. (12 points)
The air inside the bent pipe is observed to carry a *travelling* sinusoidal sound wave. Keeping in mind the result of part (**c**.), what is the lowest angular frequency ω_t that can characterize this wave? What is the ratio of ω_t to the result ω_s of part (**b**.)?

Problem 6. (30 points)
Nonviscous fluid matter is in spherically symmetric, nonrelativistic flow toward a black hole of mass M. Only the gravitational attraction of the black hole itself (as opposed to the gravitational attraction of other fluid elements) is important to the fluid motion. M is growing slowly enough to be taken as constant.

a. (10 points)
Consider Φ, the potential energy per unit mass of fluid due to the gravitational attraction of the black hole. Starting from the standard formula for the gravitational force between two point objects, show that

$$\Phi(r) = -\frac{GM}{r}$$

where r is the distance from the black hole. Note that in spherical polar coordinates

$$\nabla u = \hat{\mathbf{r}}\frac{\partial u}{\partial r} + \ldots$$

(You may use this result for Φ in subsequent parts of the problem.)

b. (10 points)
Because M is taken as constant, the fluid flow is *steady*:

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{\partial \rho}{\partial t} = 0$$

where ρ is the mass density. Also, the fluid *pressure* p is known not to vary either with position or time.

Away from the black hole, determine the dependence of fluid |velocity| v upon r.

c. (10 points)
Away from the black hole, determine the dependence of fluid mass density ρ upon r. Note that, in spherical polar coordinates,

$$\nabla \cdot \mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \ldots$$